# Math 4500 HW #13 Solutions

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*This solution set is not error-free. Please email me (gl479@cornell.edu) if you spot any errors or typos!*

**Problem 1** (Exercise 9.3.1 (10 pts))**.** Using the fact that $\Phi$ is a group homomorphism, show that we also have $(\Phi \circ C)'(0) = (\Phi \circ A)'(0) + (\Phi \circ B)'(0)$ where $C(t) = A(t)B(t)$.

*Solution.* Since $\Phi$ is a homomorphism, we have

$$\begin{aligned}(\Phi \circ C(t))'(0) &= (\Phi \circ A(t)B(t))'(0)\\ &= ((\Phi \circ A(t))(\Phi \circ B(t)))'(0)\\ &= (\Phi \circ A(t))'(0)(\Phi \circ B(t))(0) + (\Phi \circ A(t))(0)(\Phi \circ B(t))'(0)\\ &= (\Phi \circ A(t))'(0) + (\Phi \circ B(t))'(0).\end{aligned}$$

□

**Problem 2** (Exercise 9.3.2 (5 pts))**.** Deduce from Exercise 9.3.1 that $\varphi(A'(0) + B'(0)) = \varphi(A'(0)) + \varphi(B'(0))$.

*Solution.* Let $C(t) = A(t)B(t)$, then $C'(t) = A'(t) + B'(t)$. By previous problem, we are done. □

**Problem 3** (Exercise 9.3.3 (10 pts))**.** Let $D(t) = A(rt)$ for some real number $r$. Show that $D'(0) = rA'(0)$ and $(\Phi \circ D)'(0) = r(\Phi \circ A)'(0)$.

*Solution.* By the chain rule,

$$D'(t) = (A(rt))' = rA'(rt),$$

hence $D'(0) = rA'(0)$. On the other hand, let $\Psi := \Phi \circ A$, then

$$\begin{aligned}(\Phi \circ D(t))'(0) &= (\Psi \circ rt))'(0)\\ &= r(\Psi \circ t))'(0) = rD'(0).\end{aligned}$$

□

**Problem 4** (Exercise 9.3.4 (5 pts))**.** Deduce from Exercise 9.3.2 and 9.3.3 that $\varphi$ is linear.

*Solution.* Directly from previous exercises. □

**Problem 5.** Classify all the 2-dimensional Lie algebras and find their groups.

*Solution.* $\mathbb{C}^2$ with trivial Lie bracket, and its group can be $\mathbb{C}^2$. The other one is $\mathbb{C}^2$ with Lie bracket defined in Page 89. Its group is $\mathrm{Aff}(1)$. □

**Problem 6** (Exercise 8.7.3 (0 pts))**.**

*Solution.* □

**Problem 7** (Exercise 8.7.4 (0 pts))**.**

*Solution.* □

**Problem 8** (Exercise 8.7.5 (0 pts))**.**

*Solution.* □